

Relativistic Fock Space Coupled Cluster beyond CCSD: Theory and Implementation

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Current problems of the FS-RCC theory

Relativistic Fock space coupled cluster method (FS-RCC)

seems to be one of the most perspective tools for high-precision electronic structure modelling of heavy atoms and molecules

+/- solved problems:

- transition properties (e.g. intensities) → finite-field technique*
- intruder states problem → IH / denominator shifts & extrapolations**

open problems:

- only the FS-CCSD approximation is available now (e.g. DIRAC, TRAFS-3C)
- narrow scope of applicability: max 2 open shells

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**How to extend the scope of applicability
and increase accuracy of FS-RCC?**

* A. Zaitsevskii et al. Opt. Spectrosc., 124, 451 (2018)

** A. Zaitsevskii et al. PRA, 96, 022516 (2017); A. Zaitsevskii, E. Eliav. IJQC, 118, e25772 (2018)

FS-RCC Ansatz and working equations

- Wave operator:

$$\Omega = \{\exp(T)\}_N$$

CCSD: $T = T_1 + T_2$

CCSDT: $T = T_1 + T_2 + T_3$

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- Amplitude equations:

$$[T^{(h,p)}, H_0] = (V\Omega - \Omega(V\Omega)_{cl})_{conn}^{(h,p)}$$

complex combination
of integrals and
amplitudes

- Effective Hamiltonian:

$M^{h+p} \times M^{h+p}$ matrix,
 M – active space dim



$$H_{eff}^{(h,p)} = P^{(h,p)} H \{e^T\} P^{(h,p)}$$

projection onto
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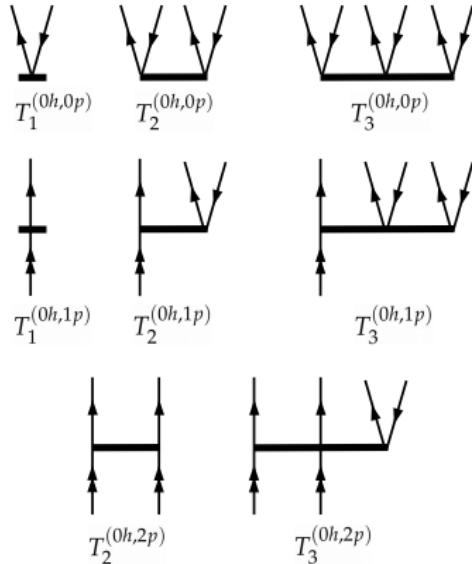
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we will work with Brandow diagrams

FS-RCC cluster operator: CCSDT approximation

example: the (0h,2p) sector



- CCSD: no spectator triples = no differential correlation
- model space extension actually does not recover all triples contributions

Example: Pb atom EEs, IH-FS-CCSD gives errors of order $200 - 600 \text{ cm}^{-1}$ *

* A. Landau, E. Eliav, Y. Ishikawa, U. Kaldor, JCP 114, 2977 (2001)

Full CCSDT: the achievable ideal?

Connected $T_3^{(h,p)}$ amplitudes can be defined for all FS sectors up to $h + p = 3$

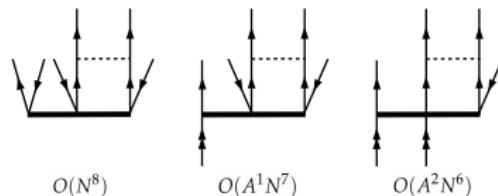
- similar to the FS-CCSD model:
 $h + p \leq 3 \rightarrow$ iterative solution
 $h + p < 3 \rightarrow$ non-iterative construction of H_{eff}
- very high accuracy
→ remember about incomplete basis, QED, Breit, ...
- strongly required for high-precision calculations in the (0h,3p) sector

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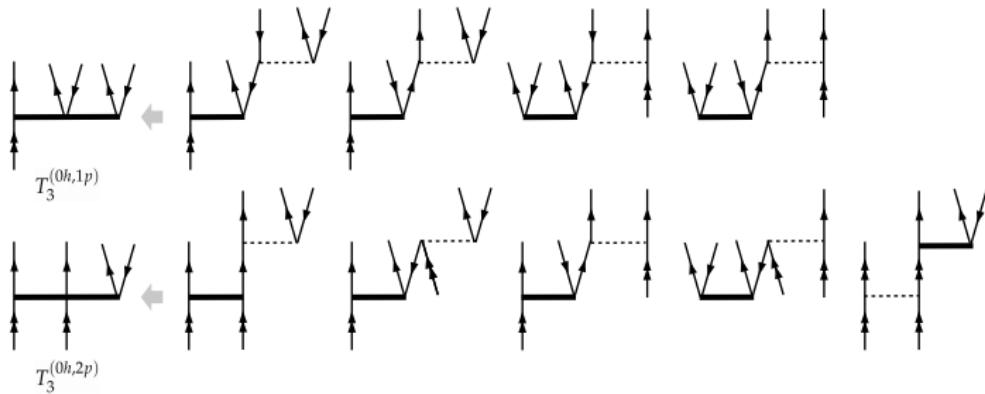
Main problem:



The CCSD+T(3) model

Let us try to estimate $T_3^{(h,p)}$ amplitudes using MBPT arguments

T_3 amplitudes are estimated only once, converged T_1 and T_2 amplitudes are used

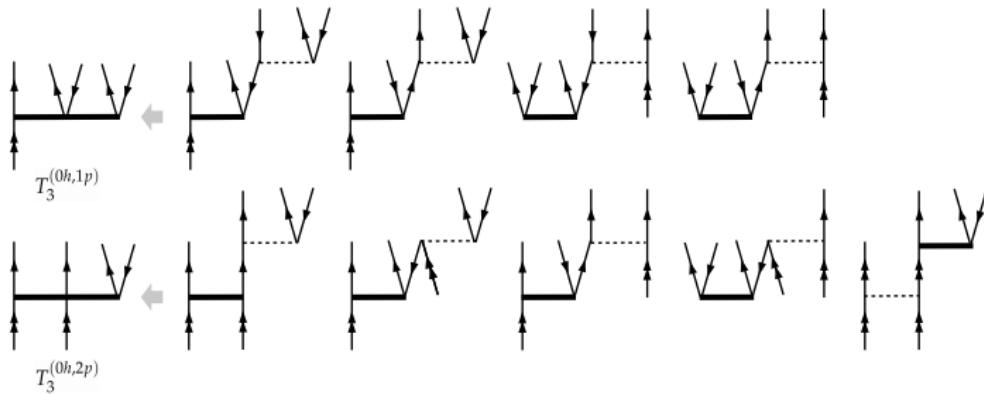


- only diagrams appearing in 3rd PT order contribute to H_{eff}
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Main drawback: works *very poorly* except (0h,0p) [Bernholdt, Bartlett, 1999]

S.R. Hughes, U. Kaldor, CPL 204, 339 (1993)

A. Oleynichenko, A. Zaitsevskii, L. V. Skripnikov, E. Eliav, Symmetry, 2020 submitted to Symmetry

CCSDT-n models

Within the CCSDT-n framework $T_3^{(h,p)}$ amplitudes contribute to T_1 and T_2 equations

CCSDT-1:

$$\left. \begin{aligned} [T_3, H_0] &\approx VT_2 \\ T_1, T_2 &\leftarrow f(V, T_1, T_2, T_3) \end{aligned} \right\} \text{repeat until convergence}$$

- iterative solution only in low FS sectors ($h + p \leq 2$)
- computational complexity is lower than for full CCSDT
- $T_3^{(h,p)}$ are to be recalculated at each iteration

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CCSDT-2, CCSDT-3:

- additional terms including T_1 and T_2 contribute to $T_3^{(h,p)}$
- $O(N^8)$ -terms are again avoided

SR-CCSDT-n: J. Noga, R. Bartlett, M. Urban. CPL, 134, 126 (1987)

FS-CCSDT-1: S. R. Hughes, U. Kaldor. CPL, 194, 99 (1992)

modern implementation is required

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the EXP-T program system

The EXP-T program system: efficient and flexible implementation of FS-RCC models

CC models:

- ✓ CCSD
- ✓ CCSD+T(3)
- ✓ CCSDT-1,2,3
- ✓ CCSDT

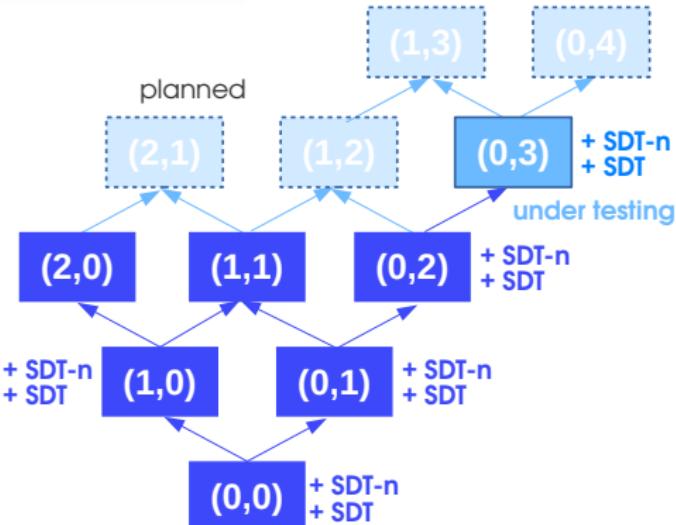
Transformed integrals:

- ✓ DIRAC
- ✓ any abelian groups
- ✓ 4cDC, X2Cmmf, 2cECP
- ✓ **Gaunt, properties**
(D. Maison, L. Skripnikov, PNPI)

Parallelization:

- ✓ OpenMP
- ✓ CUDA

$$\Omega = \{\exp(\mathbf{T})\}$$



(h,p) – Fock Space sectors

Coming soon:

<http://qchem.pnpi.spb.ru/ru/Oleynichenko>

The EXP-T program system: technical details

A. Oleynichenko, A. Zaitsevskii, E. Eliav, *Towards High Performance Relativistic Electronic Structure Modelling: The EXP-T Program Package*
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idea from the CC codes of U. Kaldor, E. Eliav and co-workers
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- **storage of symmetry and/or formal blocks of tensors: RAM, disk, disk+LZ4**



Pilot applications: EEs and IPs of TI and Pb

Additive scheme:

$$E_{CCSDT} = E_{CCSD,LB} + (E_{CCSDT,SB} - E_{CCSD,SB})$$

Triples contributions are evaluated using smaller basis set

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all-electron DCB calculations (TRAFS-3C)

QED effective potential of the Shabaev group

exhaustive basis sets

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semilocal shape-consistent 2c-ECPs of N. S. Mosyagin (PNPI), 60e in core

ANO-type contracted basis sets

Tl: $6s7p5d5f4g3h2i$, Pb: $6s6p5d5f4g3h2i$

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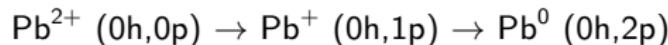
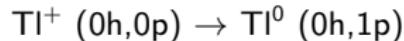
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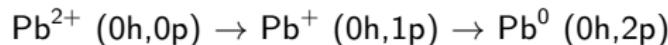
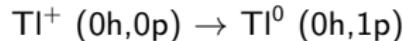
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Active space: 6p-spinors only

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A. Oleynichenko, A. Zaitsevskii, L. V. Skripnikov, E. Eliav

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2020, submitted to *Symmetry*

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Table 1. Deviations of the calculated ionization potentials (IP) and excitation energies (EE) of neutral thallium and lead and lead cation (cm^{-1}) from the experimental values. FS-RCCSD/LB+T/SB stands for the combined scheme (8).

State		Exptl [80]	IH-FS- RCCSD[47]	FS- RCCSD/LB	SDT-1	FS-RCCSD/LB + T/SB	SDT-1'	SDT-2	SDT-3	SDT
Tl, ground state $6s^2 6p\ ^2P_{1/2}$										
IP		49266		-56	-38	-38	-204	-151	-32	
EE	$6s^2 6p\ ^2P_{3/2}$	7793		-112	23	23	1	9	-31	
Pb^+ , ground state $6s^2 6p\ ^2P_{1/2}$										
IP		121245		-168	-143	-28	-190	-158	-59	
EE	$6s^2 6p\ ^2P_{3/2}$	14081		-196	-136	25	25	12	14	-42
Pb, ground state $6s^2 6p^2\ ^3P_0$										
IP		59819		-543	364	-44	-285	-347	-336	7
EE	$6s^2 6p^2\ ^3P_1$	7819		-288	-302	76	5	-4	-3	-28
	3P_2	10650		-343	-235	130	129	97	102	13
	1D_2	21458		-605	-394	215	203	158	167	5
	1S_0	29467		-208	414	170	248	293	302	173

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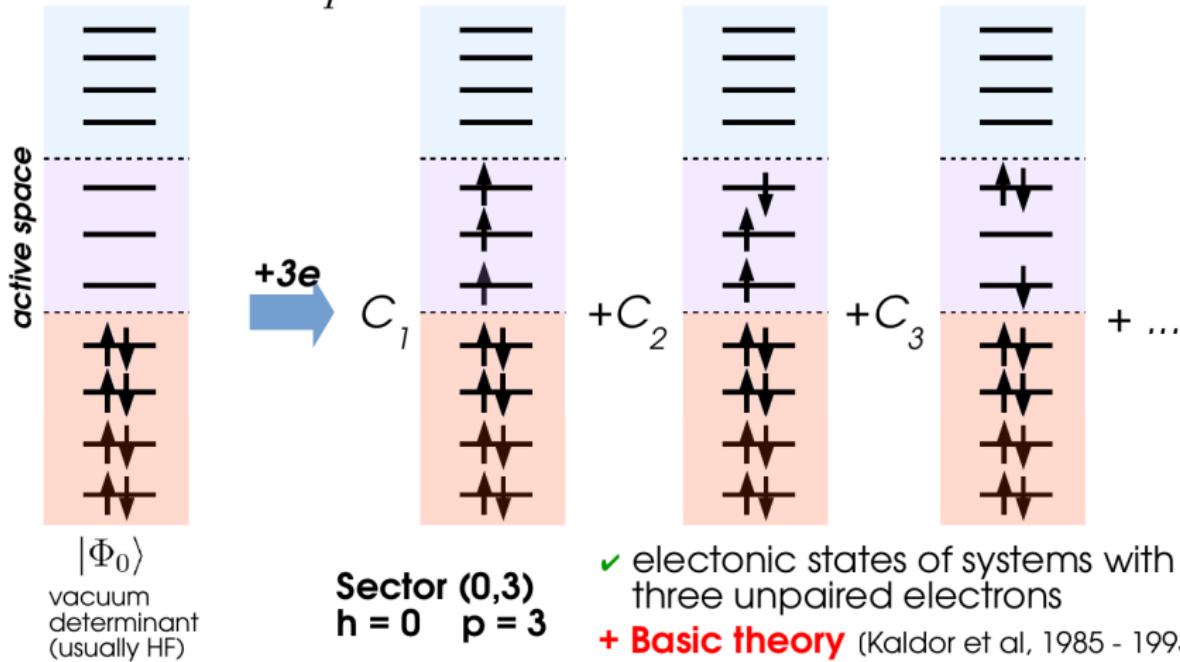
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the most accurate *ab initio* results for heavy non-alkali atoms

the (0h,3p) Fock space sector

The (0h,3p) FS sector: three particles over vacuum

$$|\psi_x\rangle = \{e^T\} \sum_I C_I |\Phi_I\rangle \quad \text{"High sectors" of the Fock space}$$



- ✓ electronic states of systems with three unpaired electrons
- + **Basic theory** (Kaldor et al, 1985 - 1993)
- **No working code !**
- **No experience !**

The (0h,3p) FS sector: motivation

Protactinium: atomic energy levels

reference state: $\text{Pa}^{3+}([\text{Rn}]7s^2)$

	Configuration	Term	J	Level (cm^{-1})
0h3p	$5f^2(^3\text{H})6d7s^2$	${}^4\text{K}$	11/2	0.000
			13/2	3711.625
			15/2	7512.695
			17/2	11198.270
0h3p	$5f^2(^3\text{H})6d7s^2$	${}^4\text{I}$	9/2	825.415
			11/2	4121.450
			13/2	7383.295
			15/2	10049.875
0h3p	$5f^26d7s^2$	$a\ {}^4\text{G}$	5/2	1618.325
			7/2	4713.870
			9/2	7330.815
0h3p	$5f(^2\text{F})6d^2(^3\text{F})7s^2$	${}^4\text{H}^\circ$	7/2	1978.220
			9/2	5335.730
			11/2	8419.075
			13/2	11498.725
0h3p	$5f(^2\text{F})6d^2(^3\text{F})7s^2$	${}^4\text{T}^\circ$	9/2	2659.405

- electronic states can be described **only in the (0h,3p) sector**
- EOM-CC cannot deal with such electronic states *at all*
- other examples: LiSr dimer, Np, Pu compounds, superactinides atoms

The (0h,3p) FS sector: first tests

Almost all electronic states of the **nitrogen atom** can be represented as three particles over the N^3+ ($1s^22s^2$) vacuum

All electrons were correlated, cc-pVTZ basis set

FSCC: $N^3+ \rightarrow N^{2+}$ (0h,1p) $\rightarrow N^+$ (0h,2p) $\rightarrow N^0$ (0h,3p)

Active space: 2p

Ground state: ${}^4S^o$ ($2s^22p^3$)

<i>(deviations from FCI, cm⁻¹)</i>					
	FCI, cm ⁻¹	TEA-EOM-CCSD*	FS-CCSD	FS-CCSDT	
${}^2D^o$	$2s^22p^3$	20275	-1175	-846	-76
${}^2P^o$	$2s^22p^3$	30038	-6738	-1134	+452

* M. Musial et al, J. Chem. Phys. 137, 174102 (2012)

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to be tested and understood

Ongoing projects

- molecular applications
- high sectors
- MPI parallelization
- triples for properties (with L. V. Skripnikov)
- first public release of EXP-T

Acknowledgements

Thank you for your attention!

thanks to

Anastasia Borschevsky

Timur Isaev

Sergey Kozlov

Leonid Skripnikov

Andrey Stolyarov

Anatoly Titov

Lucas Visscher

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